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The chirally-odd twist-3 distribution $e^a(x)$

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Abstract: Properties of the nucleon twist-3 distribution function $e^a(x)$ are reviewed. It is emphasized that the QCD equations of motion imply the existence of a δ-function at $x = 0$ in $e^a(x)$, which gives rise to the pion-nucleon sigma-term. According to the resulting "practical" DIS sum rules the first and the second moment of $e^a(x)$ vanish, a situation analogue to that of the pure twist-3 distribution function $\overline{g}_2(x)$.

Keywords: QCD, Sum Rules, Deep Inelastic Scattering, Phenomenological Models.

Contents

1. Introduction

Among the six distribution functions $f_1^a(x)$, $g_1^a(x)$, $h_1^a(x)$ and $g_T^a(x)$, $h_L^a(x)$, $e^a(x)$, which describe the structure of the nucleon in deeply inelastic scattering (DIS) processes up to twist-3, the least known and studied one is probably $e^a(x)$ (we use throughout the notation of refs. [1, 2]). The distribution function $e^a(x)$ is twist-3 and chirally odd. Apart from the first moment of $(e^u + e^d)(x)$, which sometimes is related to the phenomenologically most interesting pion-nucleon sigma-term, it is experimentally unknown. Only recently it became clear how $e^a(x)$ — in principle — could be accessed in DIS experiments. Most recently the corresponding process has been studied by the HERMES and CLAS collaborations. In particular, the CLAS data possibly provide the first experimental indications for $e^a(x)$.

This note has partly the character of a brief review, however, also some new results are reported. In section 2 the definition of $e^a(x)$ is given, and its theoretical properties are discussed. The known but only casually mentioned fact is emphasized, that $e^a(x)$ contains a $\delta(x)$-contribution. Different statements in literature on the small-x behaviour of $e^a(x)$, which at first glance seem to be contradictory, are shown to be consistent. Sum rules for $e^a(x)$ are discussed. It is argued that there is no twist-3 inequality constraining $e^a(x)$ in terms of other twist-3 distribution functions. In section 3 a brief overview is given about model calculations of $e^a(x)$. In particular results from the non-relativistic model, bag model, spectator model and chiral quark-soliton model are discussed. Section 4 briefly reports the recent progress on understanding time-odd phenomena — in particular in the fragmentation processes — which give rise to single spin asymmetries. Such asymmetries have recently been studied by the HERMES and CLAS collaborations. Section 5 contains the summary and conclusions. Some technical details concerning the gauge invariant decomposition of $e^a(x)$ can be found in appendix A.

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2. $e^q(x)$ in theory

Definition. The chirally odd twist-3 distribution functions $e^q(x)$ and $e^{\bar{q}}(x)$ for quarks of flavour q and antiquarks of flavour $\bar{q}$ are defined as [1, 2]

$$e^q(x) = \frac{1}{2M_{\rm N}} \int \frac{d\lambda}{2\pi}\, e^{i\lambda x} \langle N|\, \bar{\psi}_q(0)\, [0,\lambda n]\, \psi_q(\lambda n)\, |N\rangle\, , \qquad e^{\bar{q}}(x) = e^q(-x)\, , \tag{2.1}$$

where $[0,\lambda n]$ denotes the gauge-link. The scale dependence is not indicated for brevity. The light-like vectors n^μ in eq. (2.1) and p^μ are defined such that $n^\mu p_\mu = 1$ and the nucleon momentum is given by $P_{\rm N}^\mu = n^\mu + \frac{1}{2}M_{\rm N}^2 p^\mu$. The matrix element in eq. (2.1) is averaged over nucleon spin, i.e. $\langle N|\dots|N\rangle \equiv \frac{1}{2}\sum_{S_3}\langle N,S_3|\dots|N,S_3\rangle$.

Evolution. The renormalization scale dependence of $e^a(x)$ has been studied in refs. [3, 4, 5], see also refs. [6, 7] for reviews. The evolution of $e^a(x)$ is characterized by a complicated operator mixing pattern typical for twist-3 quantities. In the multi-colour limit the evolution of $e^a(x)$ simplifies to a DGLAP-type evolution — as it does for the other two nucleon twist-3 distribution functions $h_L^a(x)$ and (the flavour non-singlet) $g_T^a(x)$.

Sum rules for the 1st and 2nd moment. The first moment of $(e^u+e^d)(x)$ is related to the pion-nucleon sigma-term [2]

$$\int_{-1}^{1} dx\, (e^u+e^d)(x) = \frac{1}{2M_{\rm N}}\langle N|\,\left(\bar{\psi}_u\psi_u + \bar{\psi}_d\psi_d\right)\,|N\rangle \equiv \frac{\sigma_{\pi\rm N}}{m}\, , \tag{2.2}$$

$m \equiv \frac{1}{2}(m_u+m_d)$ is the average mass of the light quarks. The pion-nucleon sigma-term $\sigma_{\pi\rm N}$ [8] is defined as the value of the nucleon scalar isoscalar form factor $\sigma(t)$,

$$\sigma(t) = \frac{m}{2M_{\rm N}}\langle N(P')|\,\left(\bar{\psi}_u\psi_u + \bar{\psi}_d\psi_d\right)\,|N(P)\rangle\, , \qquad t=(P-P')^2\, , \tag{2.3}$$

at $t=0$, i.e. $\sigma_{\pi\rm N}\equiv\sigma(0)$.[1] The relation (2.2) of $e^q(x)$ to $\sigma_{\pi\rm N}$ is correct in a formal mathematical sense. The sum rule (2.2), however, unfortunately is of *no practical use* (not even in principle) to gain information on $\sigma_{\pi\rm N}$ from DIS experiments — as we shall see below.

The pion-nucleon sigma-term $\sigma_{\pi\rm N}$ gives the amount by which the nucleon mass changes, when the u- and d-quarks are given a small mass m [8]. The form factor $\sigma(t)$ describes the elastic scattering off the nucleon due to the exchange of an isoscalar spin-zero particle, and is not known experimentally except for its value at the Cheng-Dashen point $t=2m_\pi^2$. Low energy theorems [9] allow to relate $\sigma(2m_\pi^2)$ to pion nucleon scattering amplitudes and one finds

$$\sigma(2m_\pi^2) = \begin{cases} (64\pm 8)\,\text{MeV} & \text{ref. [10]} \\ (79\pm 7)\,\text{MeV} & \text{ref. [11]}\, . \end{cases} \tag{2.4}$$

[1] More precisely $\sigma(t)$ is defined as the nucleon form factor of the double commutator of the strong interaction hamiltonian with two axial isovector charges [8]. In the definition (2.3) a "double isospin violating term" proportional to $(m_u-m_d)(\bar{\psi}_u\psi_u-\bar{\psi}_d\psi_d)$ is neglected.

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The difference $\sigma(2m_\pi^2) - \sigma(0)$ has been calculated from a dispersion relation analysis [12] and in chiral perturbation theory [13] with the consistent result

$$\sigma(2m_\pi^2) - \sigma(0) \simeq 14 \text{ MeV}\,. \tag{2.5}$$

This means that

$$\sigma_{\pi\mathrm{N}} = \sigma(0) \simeq (50 - 65)\,\text{MeV}\,. \tag{2.6}$$

With $m \simeq 7\,\text{MeV}$ (at a scale of say $1\,\text{GeV}^2$) one concludes a large number for the first moment of $(e^u + e^d)(x)$

$$\int_{-1}^{1} dx\ (e^u + e^d)(x) \simeq (6 - 10)\,. \tag{2.7}$$

Since $\sigma_{\pi\mathrm{N}}$ is normalization scale invariant while the running current quark masses decrease with increasing scale, the number in eq. (2.7) becomes even larger at higher scales.

The second moment of $e^q(x)$ is proportional to the number of the respective valence quarks N_q (for proton $N_u = 2$ and $N_d = 1$) and vanishes in the chiral limit [2]

$$\int_{-1}^{1} dx\ x\, e^q(x) = \frac{m_q}{M_\mathrm{N}}\ N_q\,. \tag{2.8}$$

Twist-3 inequality. In ref. [14] the "Soffer-inequality" for twist-2 nucleon distribution functions,

$$f_1^a(x) + g_1^a(x) \geq 2|h_1^a(x)|\,, \tag{2.9}$$

was obtained making use of the positivity of the scattering density matrix. (An alternative derivation was given shortly after in [15].) Assuming that this argument can be generalized to higher twists, similar inequalities were obtained for twist-3 and twist-4 distribution functions in [14]. In particular, the "twist-3 Soffer inequality" reads $e^a(x) + h_L^a(x) \geq 2|g_T^a(x)|$. Unfortunately, in general the positivity argument is not valid for higher twists.

One way to understand this is to recall that the twist-2 inequalities are derived by relating the imaginary part of the elastic forward quark-nucleon scattering amplitude by means of the optical theorem to the total cross section (which is positive). In the Bjorken-limit the imaginary part of the amplitude can be expressed in terms of twist-2 parton distribution functions. Twist-3 effects, of course, can be taken into account. They appear as corrections of the order M_N/Q to the imaginary part of the amplitude. There is, however, in general no reason for such corrections to be positive. In other words, it is not possible to impose positivity at the level of each twist separately, fore the positivity of the scattering density matrix is already guaranteed by the twist-2 distribution functions (in the limit of large $Q \gg M_\mathrm{N}$). Interestingly, if for some reason the twist-2 inequality (2.9) is saturated, i.e. if $f_1^a(x) + g_1^a(x) = 2|h_1^a(x)|$, then the positivity of the cross section requires certain twist-3 inequalities to hold, and in one case $e^a(x) + h_L^a(x) \geq 2|g_T^a(x)|$ holds.

A different argument why twist-3 inequalities generally fail was given in ref. [15]. In the framework of light-cone formalism it was demonstrated that possible positivity constraints on the twist-3 distributions $e^a(x)$, $h_L^a(x)$ and $g_T^a(x)$ inevitably involve twist-2 *and* twist-4 distributions — which makes such constraints practically useless [15]. (Still, in some cases useful constraints — though involving different twists — can be obtained, see [16].)

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The large N_c limit. In the limit of a large number of colours N_c one observes the following behaviour of the singlet and non-singlet flavour combinations [17]

$$\begin{aligned}(e^u + e^d)(x) &= N_c^2 d(N_c x)\\ (e^u - e^d)(x) &= N_c\, d(N_c x)\,,\end{aligned} \tag{2.10}$$

where the functions $d(y)$ are stable in the large-N_c limit for a fixed argument $y = N_c x$, and of course different for the different flavour combinations. From (2.10) we conclude that

$$|(e^u + e^d)(x)| \gg |(e^u - e^d)(x)|x \quad \text{for } N_c \to \infty\,, \tag{2.11}$$

or in other words $e^u(x) \approx e^d(x)$ modulo $1/N_c$-corrections. Such large-N_c relations hold well in nature, see e.g. ref. [18], and can serve as a useful guideline.

Decomposing $e^a(x)$ by means of the QCD equations of motion. The following operator identity follows from the QCD-equations of motion (flavour indices on the quark fields are omitted for simplicity)

$$\begin{aligned}\bar\psi(0)[0,z]\psi(z) = {} & \bar\psi(0)\psi(0)+\\ & +\frac{1}{2}\int_0^1 du \int_0^u dv \bar\psi(0)\sigma^{\alpha\beta}z_\beta[0,vz]gG_{\alpha\nu}(vz)z^\nu[vz,uz]\,\psi(uz)-\\ & -im_q\int_0^1 du\bar\psi(0)\not{z}[0,uz]\,\psi(uz)-\\ & -\frac{i}{2}\int_0^1 du\Big(\bar\psi(0)(i\not{D}-m_q)\not{z}[0,uz]\psi(uz)+\bar\psi(0)\not{z}[0,uz](i\not{D}-m_q)\psi(uz)\Big).\end{aligned} \tag{2.12}$$

The identity (2.12) is exact up to total derivatives which are irrelevant for the parton distribution functions. The formalism to derive such identities has been introduced in ref. [19]. The identity (2.12) can be found, e.g., in refs. [4, 7]. The "equations of motion" operator in the last line of eq. (2.12) vanishes in physical matrix elements, however, its mixing under renormalization with the other operators in eq. (2.12) has to be considered in the study of the evolution properties of $e^a(x)$, see [3, 4, 5] and [6, 7]. The other operators in the identity (2.12), after inserted into the definition (2.1), yield the following decomposition of $e^q(x)$

$$e^q(x) = e^q_{\rm sing}(x) + e^q_{\rm tw3}(x) + e^q_{\rm mass}(x)\,. \tag{2.13}$$

The contributions $e^q_{\rm sing}(x)$, $e^q_{\rm tw3}(x)$ and $e^q_{\rm mass}(x)$ are "physically real", in the sense that each term in the operator decomposition in eq. (2.12) is gauge-invariant. They are defined as follows (see also appendix A).

The contribution $e^q_{\rm sing}(x)$ arises from the local scalar operator $\bar\psi_q(0)\psi_q(0)$ on the right-hand-side (r.h.s.) of the identity (2.12). It is proportional to a δ-function at $x = 0$

$$e^q_{\rm sing}(x) = \frac{\delta(x)}{2M_{\rm N}}\,\langle N|\bar\psi_q(0)\psi_q(0)|N\rangle\,. \tag{2.14}$$

The presence of this singular term is well known but only rarely mentioned (see, e.g., the footnote on p. 233 of ref. [7]). It is customary to cancel out this contribution by multiplying $e^q(x)$ by x, since it has no partonic interpretation and is not relevant for the discussion of the evolution properties of $e^q(x)$. However, this contribution gives rise to the pion-nucleon sigma-term [3]. The possible existence of a $\delta(x)$ singularity in $e^q(x)$ and structure functions associated with it has a long history. We shall come to this point later on.

The contribution $e^q_{\rm tw3}(x)$ is a quark-antiquark-gluon correlation function, i.e. the actual "pure" twist-3, "interaction dependent" contribution to $e^q(x)$. It has a "partonic interpretation" as an interference between scattering from a coherent quark-gluon pair and from a single quark, see [1, 2] and references therein. It is due to the second operator on the r.h.s. of the identity (2.12). The explicit expression of this term can be found in [2, 3, 4, 5] and [6, 7] (see also appendix A). Here we only mention that the first two moments of $e^q_{\rm tw3}(x)$ vanish, i.e.

$$\mathcal{M}_n[e^q_{\rm tw3}] = \int_{-1}^{1} dx\; x^{n-1}\, e^q_{\rm tw3}(x) = 0 \quad \text{for } n = 1,\, 2\,. \tag{2.15}$$

The contribution $e^q_{\rm mass}(x)$ is proportional to the current quark mass, and is conveniently defined in terms of its Mellin moments

$$\mathcal{M}_n[e^q_{\rm mass}] = \frac{m_q}{M_{\rm N}} \times \begin{cases} 0 & \text{for } n = 1, \\ \mathcal{M}_{n-1}[f_1^q] & \text{for } n > 1, \end{cases} \tag{2.16}$$

where $\mathcal{M}_n[f_1^q]$ are the moments of the twist-2 unpolarized distribution $f_1^q(x)$. The relation (2.16) for moments $n \geq 2$ can be inverted as

$$x\, e^q_{\rm mass}(x) = \frac{m_q}{M_{\rm N}}\, f_1^q(x)\,. \tag{2.17}$$

Of course, for $x \neq 0$ one can divide eq. (2.17) by x — this allows to draw conclusions on the behaviour of $e^q(x)$ at small $x > 0$ (see below). However, one has to keep in mind that the correct definition of $e^q_{\rm mass}(x)$ is eq. (2.16).

Small-x behaviour of $e^q(x)$. The small- and large-x behaviour of pure twist-3 distributions has been studied, see e.g. eqs. (6.38, 6.39) in ref. [7]. For the following discussion we note the result

$$e^q_{\rm tw3}(x) \to \text{const}\; x^0 \quad \text{for } x \to 0\,. \tag{2.18}$$

Let us consider small $x \neq 0$ and the chiral limit ($m_q \to 0$), where the mass-term in eq. (2.17) drops out. Then eq. (2.18) dictates the small-x behaviour of $e^q(x)$, and we have $e^q(x) \to$ const for small but non-zero x. This result is consistent with the conclusion

$$e^q(x) \to \text{const}\; x^{-0.04} \quad \text{for } x \to 0\,, \tag{2.19}$$

drawn in ref. [20] from Regge phenomenology assuming a linear trajectory with standard slope (because the Regge trajectory could be slightly non-linear or its slope could be slightly different from $1\,\mathrm{GeV}^{-2}$ and have an intercept $\alpha(0) = -1$). In particular, in [20] it was concluded that the Pomeron decouples, because the Pomeron residue is spin-non-flip.

Considering finite quark mass effects, however, we see that the behaviour of $e^q(x)$ for small but non-zero x is dominated by the mass-term $e^q_{\rm mass}(x) = m_q f_1^q(x)/(M_{\rm N}x)$ in eq. (2.17), and that the Pomeron contributes to $e^q(x)$ (since it contributes to $f_1^q(x)$). This agrees with the conclusion

$$e^q(x) \to \text{const}\,\frac{1}{x^2} \quad \text{for } x \to 0\,, \tag{2.20}$$

drawn in ref. [2] from Regge phenomenology, where it was argued that the "Pomeron couples", even though suppressed by the factor $m_q/M_{\rm N}$.

The small x behaviour of $e^q(x)$ in (2.20) makes — at first glance — questionable the convergence of the sum rule (2.2) [2]. However, from eq. (2.16) we see that $e^q_{\rm mass}(x)$ does not contribute to (2.2). Thus it is not the small x behaviour in (2.20) which makes the sum rule (2.2) practically useless for the purpose of relating $e^q(x)$, as it could be measured in DIS, to the pion nucleon sigma term. Rather it is, of course, the $\delta(x)$ contribution.

Conclusions from the use of equations of motion. The first conclusion is that the pion nucleon sigma term originates from the singular $\delta(x)$-contribution $e^q_{\rm sing}(x)$ only. This follows from comparing eqs. (2.2) and (2.14) and considering eqs. (2.15), (2.16), i.e.

$$\int_{-1}^{1} dx\; e^q(x) = \int_{-1}^{1} dx\; e^q_{\rm sing}(x) = \frac{1}{2M_{\rm N}}\,\langle N|\bar\psi_q(0)\psi_q(0)|N\rangle\,. \tag{2.21}$$

If one neglects current quark mass effects[2] eq. (2.21) has the following consequence. Recalling that $e^{\bar q}(x) = e^q(-x)$ and integrating over x in the interval $[0_+,1] \equiv [\epsilon,1]$ with a positive ϵ arbitrarily close (but not equal) to zero, one obtains

$$\int_{0+}^{1} dx\; (e^q + e^{\bar q})(x) = 0\,. \tag{2.22}$$

The existence of the $\delta(x)$, of course, cannot be confirmed in the experiment. Eq. (2.22), however, corresponds to the experimental situation and could in principle be tested in the experiment.

Finite current quark mass effects. Neither the pure twist-3 contribution $e^q_{\rm tw3}(x)$ (due to eq. (2.15)) nor the singular term $e^q_{\rm sing}(x)$ (due to $\int_{-1}^1 dx\; x\,\delta(x) = 0$) contribute to the second moment of $e^q(x)$. Thus the sum rule in eq. (2.8) is saturated by the mass term, i.e.

$$\int_0^1 dx\; x\,(e^q - e^{\bar q})(x) = \frac{m_q}{M_{\rm N}}\int_0^1 dx\;(f_1^q - f_1^{\bar q})(x) = \frac{m_q}{M_{\rm N}}\,N_q\,. \tag{2.23}$$

[2]Finite quark mass effects are considered in the next paragraph.

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Let us investigate in detail the effect of finite quark mass in eq. (2.22). The integral of $e^a_{\rm mass}(x)$ over x in $[-1,1]$ yields zero according to eq. (2.16). So for any $0 < x_{\rm min} \leq 1$ the following equation is formally true

$$\int\limits_0^{x_{\rm min}} dx\;(e^q_{\rm mass}+e^{\bar q}_{\rm mass})(x) = -\int\limits_{x_{\rm min}}^1 dx\;(e^q_{\rm mass}+e^{\bar q}_{\rm mass})(x) = -\frac{m_q}{M_{\rm N}}\int\limits_{x_{\rm min}}^1 dx\;\frac{(f_1^q+f_1^{\bar q})(x)}{x} \leq 0\,. \tag{2.24}$$

In the third integral in (2.24) we made use of eq. (2.17) divided by x (which is allowed since the point $x=0$ is not included in that integral). The final step in (2.24) follows from the positivity of $f_1^a(x)$. All integrals in (2.24) are (formally) well defined for any $x_{\rm min} > 0$.

Thus, if one does not neglect $m_q/M_{\rm N}$, the DIS sum rule (2.22) for the first moment of $e^a(x)$ becomes[3]

$$\int\limits_{x_{\rm min}}^1 dx\;(e^q+e^{\bar q})(x) = \frac{m_q}{M_{\rm N}}\int\limits_{x_{\rm min}}^1 dx\;\frac{(f_1^q+f_1^{\bar q})(x)}{x} \quad \text{for very small } x_{\rm min} > 0\,. \tag{2.25}$$

It is clear that also $e^a_{\rm mass}(x)$ contains a singularity at $x=0$. Formally this singularity can be written as a generalized distribution

$$e^q_{\rm mass}(x) = \frac{m_q}{M_{\rm N}}\left[\mathcal{P}\frac{f_1^q(x)}{x} - \delta(x)\mathcal{P}\int\limits_{-1}^1 dx'\,\frac{f_1^q(x')}{x'}\right] \stackrel{\rm or}{=} \lim_{\epsilon\to 0}\frac{m_q}{M_{\rm N}}\left[\frac{x f_1^q(x)}{x^2+\epsilon^2} - \delta(x)\int\limits_{-1}^1 dx'\,\frac{x' f_1^q(x')}{x'^2+\epsilon^2}\right], \tag{2.26}$$

where it is understood that the principal value prescription (or the limit $\epsilon \to 0$) has to be taken only after $e^q_{\rm mass}(x)$ has been inserted in an integral and integrated over. Thus — for finite m_q — there is formally yet another δ-function at $x=0$ in $e^a(x)$. This δ-function ensures the formal "convergence" (when the point $x=0$ is included) of the sum rule (2.2) by cancelling the contribution from the mass term which strongly rises with decreasing x, cf. eq. (2.20). Thus the paradoxical situation emerges that the sum rule (2.2) "practically" (since $x=0$ cannot be reached experimentally) diverges, as noticed in [2]. But "theoretically" (when the point $x=0$ is included) the sum rule (2.2) exists (and is then saturated by the $\delta(x)$ contribution in eq. (2.14)).

In principle, the small factor $m_q/M_{\rm N}$ in eq. (2.25) could be compensated by the factor $\int_{x_{\rm min}}^1 dx\,(f_1^q+f_1^{\bar q})(x)/x$ which rapidly grows with decreasing $x_{\rm min}$. Does this mean that the relation (2.25) could in principle be used to measure current quark masses in DIS? At leading twist, current quark mass effects are not observable in DIS because they are suppressed by a hard power m_q/Q and cannot be distinguished from other (possibly non-factorizing) power suppressed contributions which are generically $\mathcal{O}(\Lambda_{\rm QCD}/Q)$. The attempt to "measure" m_q by means of eq. (2.25) is also of such kind: Presuming factorization the physical contribution to an observable of the twist-3 $e^a(x)$ is accompanied by the factor $M_{\rm N}/Q$, i.e.

[3] Due to (2.15) we have $\int_{x_{\rm min}}^1 dx\,(e^q_{\rm tw3}+e^{\bar q}_{\rm tw3})(x) = -\int_0^{x_{\rm min}} dx\,(e^q_{\rm tw3}+e^{\bar q}_{\rm tw3})(x) \approx 0$ for small $x_{\rm min}$ because of the smooth behaviour of $e^a_{\rm tw3}(x)$ at small x in (2.18). This allows to safely neglect the contribution from $e^q_{\rm tw3}(x)$ to (2.25).

the effect of m_q is effectively $(M_{\rm N}/Q) \times (m_q/M_{\rm N}) = m_q/Q$. The (purely academic) question, whether m_q could be measured in this way in DIS, would be answered by thorough proofs of factorization for processes involving $e^a(x)$. Such proofs would clarify whether m_q-contributions factorize from infrared singularities (in a process-independent way). At present, no such proof exists.

It is interesting to remark that — wherever it was *assumed* that factorization holds [36, 37, 38] — it was always $x\, e^a(x)$ (and not $e^a(x)$) which contributed to the cross-section. From a practical point of view, one can thus redefine $e^a(x) \to e^a_{\rm red}(x) \equiv x\, e^a(x)$ (as indeed some authors [4, 6] do), and the discussions about $\delta(x)$-functions become superfluous. However, then one faces the interesting phenomenon, that the pion-nucleon sigma-term originates from a non-physical — namely the "minus first" — moment of the redefined $e^a_{\rm red}(x)$. The continuation to non-physical negative moments has been discussed in ref. [21].

The $\delta(x)$ singularity in $e^q(x)$. We have seen that QCD equations of motion allow to decompose $e^q(x)$ in a gauge invariant way into three contributions, one of them being proportional to a δ-function type singularity at $x = 0$. As the limiting point $x = 0$ can neither be attributed to quark nor to antiquark distribution functions this finding is more clearly expressed as

$$(e^q + e^{\bar q})(x) = \frac{\delta(x)}{2M_{\rm N}} \langle N|\bar\psi_q(0)\psi_q(0)|N\rangle + \text{regular pure twist-3} + \mathcal{O}\left(\frac{m_q}{M_{\rm N}}\right). \tag{2.27}$$

This means that the connection of $e^q(x)$ and the pion nucleon sigma term is of purely formal character; there is no experimental relation between $\sigma_{\pi\rm N}$ and $e^q(x)$ or a structure function related to it. Interestingly, this possibility was considered already in the early 1970s before the advent of QCD [22].

In order to carefully derive DIS sum rules — such as (2.2) — one uses dispersion relations to relate the (at least in principle) measurable structure functions to the imaginary part of the respective forward scattering amplitude. The latter can then further be investigated by means of the operator product expansion, which in the Bjorken limit allows to connect moments of the structure functions to matrix elements of local operators. A sum rule formally derived using the operator product expansion is valid also for the experimentally measurable structure function if the forward scattering amplitude satisfies *unsubtracted* dispersion relations. If subtraction terms — in the context of Regge phenomenology referred to as "fixed poles" [23] — have to be included to make the dispersion integral finite, then the sum rule can be spoiled. A subtraction term in the dispersion integral manifests itself as a $\delta(x)$-contribution in the structure function [24]. (Cf. [25] for a nice pedagogical exposition.) On the basis of such dispersion relation and Regge arguments it was observed in ref. [22] that the sigma term sum rule[4] (2.2) could be spoiled. More prominent examples of sum rules which could possibly be spoiled in this way are the Burkhardt-Cottingham sum rule [26] and the Gerasimov-Drell-Hearn sum rule [27].

[4]More precisely, it is the sum rule involving the structure functions $F_4(x)$ and $F_5(x)$ which are related to $e^q(x)$ and — in principle — could be measured in (anti)neutrino-nucleon DIS.

Further indications towards the presence of a $\delta(x)$-contribution in $e^q(x)$ were presented in ref. [28], where $e^q(x)$ was constructed explicitly for a one-loop dressed massive quark. Of course, the perturbative calculation of ref. [28] does not *prove* the existence of a $\delta(x)$-contribution in $e^q(x)$. But it *strongly suggests* it since one hardly can imagine a mechanism to cancel a $\delta(x)$-contribution, which appears in the leading order of some small coupling expansion, by higher order contributions. In the next section we shall see that a $\delta(x)$-contribution has recently been observed also in *non-perturbative* calculations in the framework of a realistic model of the nucleon (chiral quark-soliton model).

3. $e^q(x)$ in models

In this section we review results obtained in the non-relativistic quark model, bag model, spectator model and chiral quark-soliton model. We also will mention calculations in some toy models.

A subtle question is whether twist-3 distribution function can be described in models with no gluon degrees of freedom. However, among the most general twist-3 structures in the nucleon $e^a(x)$, $g_T^a(x)$ and $h_L^a(x)$ are distinguished inasmuch they can be expressed in terms of quark fields only, i.e. with no *explicit* gluon fields. This allows to describe these distribution functions in models with no gluon degrees of freedom, as argued in [1, 2]. However, the results of such model calculations have to be interpreted with care.

Non-relativistic quark model. The non-relativistic limit is an intuitive and, in some cases, useful guideline. We recall the popular relation $h_1^q(x) = g_1^q(x)$, which is often used to estimate effects of the transversity distribution function. (Irrespective the fact that, taken literally, the non-relativistic model yields $h_1^q(x) = g_1^q(x) = P_q\delta(x - \frac{1}{3})$ with $P_u = 4/3$ and $P_d = -1/3$.) In this paragraph $q = u$, d since there are no antiquarks in the non-relativistic limit, and $m_q = m_u = m_d$ is to be understood as the constituent mass of the light quarks, which is one third of the nucleon mass, i.e. $m_q = M_{\rm N}/3$.

In the non-relativistic limit the twist-3 quark distribution $e^q(x)$ and the unpolarized twist-2 quark distribution $f_1^q(x)$ coincide [17]

$$\lim_{\substack{\text{non}\\\text{relativistic}}} e^q(x) = \lim_{\substack{\text{non}\\\text{relativistic}}} f_1^q(x) = N_q\; \delta\left(x - \frac{1}{3}\right). \tag{3.1}$$

For the first moment the result in (3.1) yields $\int dx\,(e^u + e^d)(x) = 3$. This is the correct non-relativistic result for the sum rule (2.2), since in this limit $\sigma_{\pi\rm N} = 3m_q = M_{\rm N}$. The latter can be verified by taking the non-relativistic limit in the expression (2.3) for $\sigma_{\pi\rm N}$, or alternatively by means of the Feynman-Hellmann theorem

$$\sigma_{\pi\rm N} = m\,\frac{\partial M_{\rm N}(m)}{\partial m}\,, \tag{3.2}$$

where $m = \frac{1}{2}(m_u + m_d) = M_{\rm N}/3$ in this case. For the second moment (3.1) yields $\int dx\, e^q(x) = N_q/3$, which is the correct non-relativistic result for the sum rule (2.8) recalling that $m_q/M_{\rm N} = 1/3$.

Thus, the non-relativistic result (3.1) satisfies the QCD sum rules (2.2), (2.8). However, the results $\sigma_{\pi\mathrm{N}} = M_\mathrm{N}$ and $\int dx\,(e^u + e^d)(x) = 3$ strongly overestimate and underestimate, respectively, the phenomenological numbers in eqs. (2.6) and (2.7). In particular, one could be worried that such a large value for the pion nucleon sigma term, $\sigma_{\pi\mathrm{N}} = M_\mathrm{N}$, would imply a huge number for the strangeness content y of the nucleon, defined as

$$y = \frac{2\langle N|\,\bar\psi_s\psi_s|N\rangle}{\langle N|\,(\bar\psi_u\psi_u + \bar\psi_d\psi_d)\,|N\rangle}\,. \tag{3.3}$$

The precise role of y in estimating the contribution of the strange quark degree of freedom to the nucleon mass was discussed in [30]. In spite of the large strangeness content y the total strange quark contribution to the nucleon mass is rather small. To leading order in chiral perturbation theory the relation between y and $\sigma_{\pi\mathrm{N}}$ is given by [8]

$$y = 1 - \frac{m}{m_s - m}\,\frac{M_\Xi + M_\Sigma - 2M_\mathrm{N}}{\sigma_{\pi\mathrm{N}}}\,. \tag{3.4}$$

(With $m_s/m \simeq 25$ in eq. (3.4) one obtains $y = 1 - 26\,\mathrm{MeV}/\sigma_{\pi\mathrm{N}}$. Improved calculations in higher orders of chiral perturbation theory yield $y = 1 - (35 \pm 5)\,\mathrm{MeV}/\sigma_{\pi\mathrm{N}}$ [29]. However, for our purposes the relation (3.4) is sufficient.) Inserting the non-relativistic mass relations $M_\Xi = 2m_s + m_q$, $M_\Sigma = m_s + 2m_q$ and $M_\mathrm{N} = 3m_q$ into (3.4) one observes that the mass of the strange quarks cancels out exactly, and $y = 1 - M_\mathrm{N}/\sigma_{\pi\mathrm{N}}$, i.e. $y = 0$ with the non-relativistic result $\sigma_{\pi\mathrm{N}} = M_\mathrm{N}$. This result follows directly from (3.3) since in the non-relativistic limit $\bar\psi_s\psi_s = \psi_s^\dagger\psi_s$ is the number operator for strange quarks, which has a zero expectation value in the nucleon states.

Finally we observe that in the non-relativistic limit $e^q(x)$ is due to the mass term in (2.13) only, as one intuitively would expect. This can be seen by observing that $e^q_\mathrm{mass}(x) = (m_q/M_\mathrm{N})\,f_1^q(x)/x = f_1^q(x)$, since the $\delta(x-\frac{1}{3})$-function in (3.1) allows to replace x in the denominator by $\frac{1}{3}$ which cancels the prefactor $m_q/M_\mathrm{N} = \frac{1}{3}$. Thus, the non-relativistic "description" of $e^q(x)$ is theoretically consistent but phenomenologically not correct and, of course, not suited to provide insights into the twist-3 structure of $e^q(x)$.

Bag model. The first model studies of $e^q(x)$ have been done in the bag model [2, 31]. In figure 1 the results from ref. [2] are shown. At the low scale of that model estimated to be around 0.4 GeV the quark distribution $e^q(x)$ is of comparable magnitude as $f_1^q(x)$. (The bag model is not expected to consistently describe antiquark distribution functions, since it yields $f_1^{\bar q}(x) < 0$ in contradiction with the positivity requirement.)

In the bag model the twist-2 Soffer inequality (2.9) is saturated,[5] which is a necessary (but not sufficient) condition for the (generally incorrect) "twist-3 inequality" of ref. [14] to be valid. Indeed, it is observed that the "twist-3 Soffer-inequality" holds in the bag model and is saturated, i.e. $e^q(x) = 2g_T^q(x) - h_L^q(x)$ in the bag model [31].[6]

[5]This naturally happens in quark models with (sufficiently) simple nucleon wave functions [15].

[6]This in turn is a necessary condition for the (also generally incorrect) twist-4 inequality of ref. [14] to be valid, which again holds in the bag model and is saturated [31]. Thus, out of the three linearly independent quark distribution functions, which exist in general at each twist level in a spin $\frac{1}{2}$ hadron, in

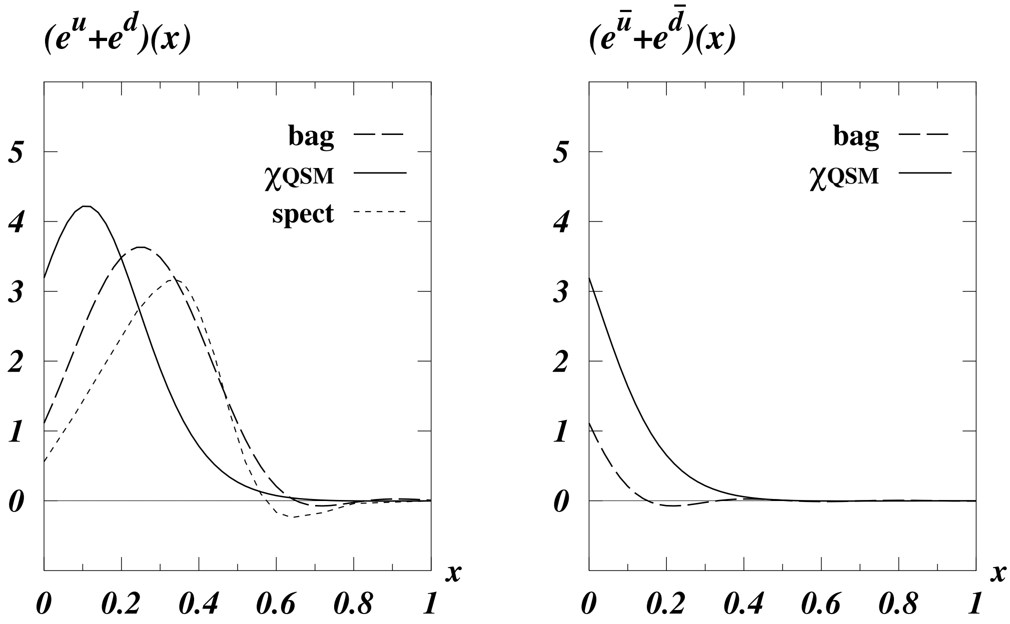

Figure 1: Left: results for the flavour singlet $(e^u + e^d)(x)$ vs. x from the bag [2], spectator [32] and chiral quark-soliton model [17]. Right: the same for anti-quarks. No attempt is made to indicate the $\delta(x)$-contribution in the chiral quark-soliton model result. The model results refer to low scales around 0.5 GeV, see text. (The flavour-independent result for "$e(x)$" from [2] is multiplied by a factor of 3 for sake of comparison to the spectator and chiral quark-soliton model results.)

There is no $\delta(x)$-contribution to $e^q(x)$ in the bag model. The first moment arises from a valence-like structure and is of the order of magnitude of unity, underestimating the phenomenologically expected number in eq. (2.7). The sum rule for the second moment in eq. (2.8) is violated in the bag model. This can be understood because this sum rule follows from equations of motion, which are modified by the bag boundary [2]. It is worthwhile mentioning that $e^q(x)$ is entirely a bag surface effect. This in some sense is consistent as the bag models confinement and thus mimics gluons [2].

Spectator model. The quark distribution functions $e^u(x)$ and $e^d(x)$ were estimated in the spectator model at a scale pressumably lower than 0.5 GeV [32]. In this model $e^u(x)$ was found sizeable and $e^d(x)$ rather small; i.e. $(e^u + e^d)(x)$ and $(e^u - e^d)(x)$ are of comparable magnitude, and the large-N_c relation (2.11) does not hold (which for finite N_c, of course, does not need to be the case). Also in the spectator model there is no $\delta(x)$-function in $e^q(x)$. The result for $(e^u + e^d)(x)$ is shown in figure 1a.

Gross-Neveu model in $(1+1)$-dimensions. In ref. [25] $e^a(x)$ was discussed in several toy models. The purpose of these studies was not to provide realistic estimates of $e^a(x)$, but to shed some light on possible mechanisms leading to the appearance of a $\delta(x)$-term.

the bag model only two respectively are linearly independent. In some sense this is analog to the situation in the non-relativistic model, where $h_1^q(x) = g_1^q(x)$. A possible explanation could be that both models contain only quarks and the longitudinally and transversely polarized nucleon states are related to each other geometrically, namely by (respectively ordinary and Melosh) rotations.

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In particular, in a *non-perturbative* calculation in the $(1+1)$-dimensional version of the Gross-Neveu model the twist-3 distribution of "s"-type (model-) quarks in the "u"-type (model-) quark $e^{s/u}(x) = \frac{1}{2M}\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle u|\bar\psi_s(0)\psi_s(\lambda n)|u\rangle$ was found to be proportional to $\delta(x)$. In the Gross-Neveu model the $\delta(x)$-term arises from long range correlations along the light cone, i.e. from the fact that the correlator $\langle u|\bar\psi_s(0)\psi_s(z)|u\rangle$ (z light-like) contains a contribution independent of z [25]. In order to illustrate that this need not be a peculiarity of the $(1+1)$-dimensional model it was shown (by means of *perturbative* methods) that $e^a(x)$ contains a $\delta(x)$-contribution also in other $(1+1)$- and $(3+1)$-dimensional field theories [25] (cf. also [28]).

Chiral quark-soliton model (χQSM). The flavour-singlet combination $(e^u+e^d)(x)$, which is the leading contribution in the large-N_c limit (2.10), has been studied in the χQSM at a low normalization point of about 0.6 GeV [17]. Interestingly, $(e^u+e^d)(x)$ was found to contain a $\delta(x)$-contribution [17]

$$(e^u+e^d)(x) = C\,\delta(x) + (e^u+e^d)(x)_{\rm reg}\,, \tag{3.5}$$

where $(e^u+e^d)(x)_{\rm reg}$ is a regular part, which has a valence-like structure and qualitatively looks similar to the bag and spectator model results, see figure 1. The coefficient C is quadratically UV-divergent in the χQSM and can consistently be regularized. It is remarkable that in the model the baryonic quantity C is proportional to the quark vacuum condensate

$$C = A_N\,\langle{\rm vac}|(\bar\psi_u\psi_u+\bar\psi_d\psi_d)|{\rm vac}\rangle\,, \qquad A_N = \frac{1}{2}\int d^3\mathbf{x}\;{\rm tr}_{\rm F}\left(\frac{U+U^\dagger}{2}-1\right). \tag{3.6}$$

The proportionality factor A_N encodes the information on the nucleon: $U=\exp(i\tau^a\pi^a)$ is the SU(2) chiral pion (soliton) field, ${\rm tr}_{\rm F}$ denotes the trace over flavour. Numerically $C=(9\pm3)$ for the value $(-280\pm30)^3{\rm MeV}^3$ of the quark-condensate from ref. [29]. In the χQSM the first moment of $(e^u+e^d)(x)$ is not solely due to the $\delta(x)$-function but also receives a (small) contribution from the regular part $(e^u+e^d)(x)_{\rm reg}$ in (3.5). The final results are $\int dx\,(e^u+e^d)(x) = (10\pm3)$ and $\sigma_{\pi{\rm N}} = 80$ MeV in reasonable agreement with eqs. (2.6), (2.7). The sum rules for the first and the second[7] moment are satisfied in the χQSM. There are no means in this model (such as gauge-invariance in QCD) to further decompose the regular part in (3.5), which is to be understood as the entangled pure-twist-3-term and mass-term.

In ref. [33] the existence of a $\delta(x)$ contribution in $(e^u+e^d)(x)$ in the χQSM was concluded independently and in an alternative way to the derivation given in [17]. In particular, in ref. [33] it was shown that the $\delta(x)$-term is due to long-distance quark-quark correlations. Thus, the underlying non-perturbative mechanism which gives rise to the $\delta(x)$ contribution in the χQSM is analog to that observed in the Gross-Neveu model [25].

[7]For the second moment, however, one cannot expect the current quark mass m_q to appear in the model-version of the QCD sum rule (2.8). Instead an "effective" mass appears because the χQSM describes *bound* quarks at a *low scale* of 0.6 GeV.

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Apparently, a $\delta(x)$-contribution has no partonic interpretation. However, the model relation (3.6) "suggests" an "intuitive understanding". Let us simplifyingly interpret $e^a(x)$ as scattering (in a particular way) off a parton in the nucleon, which carries the nucleon momentum fraction x (in the infinite momentum frame) [17]. What means scattering off a parton, which carries the momentum fraction $x = 0$? Eq. (3.6) suggests that the parton with $x = 0$ is not "moving" with the fast proton but indeed at rest. And it is not taken out of the proton but out of the vacuum, which to some extent (quantified by the constant A_N in (3.5)) is present also inside the proton [17]. It would be interesting to see whether the naive "interpretation" could be "confirmed" by observing relations analogue to (3.6) in other chiral models.

Finally we remark that the coefficient C of the $\delta(x)$-contribution in eq. (3.6) implies a relation between the pion nucleon sigma term and the quark vacuum condensate

$$\sigma_{\pi\mathrm{N}} = m \, \langle \mathrm{vac}|(\bar{\psi}_u\psi_u + \bar{\psi}_d\psi_d)|\mathrm{vac}\rangle \, A_N \tag{3.7}$$

with A_N (which is negative) as defined in eq. (3.6). The only model in which such a relation was known so far is the Skyrme model [34].[8]

Summary of model results. In the non-relativistic limit $e^a(x)$ and $f_1^a(x)$ become equal. The more realistic bag, spectator and chiral quark-soliton model [2, 17, 31, 32] suggest that $e^a(x)$ has a sizeable valence-like structure at a low scale and is roughly half the magnitude of $f_1^a(x)$. The equations of motions are modified in these models (compared to QCD) and there is no gauge principle. Therefore a decomposition analogue to (2.13) is not possible. Still, in the χQSM there is a $\delta(x)$-contribution.

The model results certainly do not discourage measurements of observables containing information on $e^q(x)$. However, as will be discussed in the next section, this is a difficult task and only recently progress has been reported.

4. $e^q(x)$ in experiments

The distribution function $e^a(x)$ is a "spin-average" distribution, i.e. accessible in experiments with unpolarized nucleons. However, due to its chiral odd nature it can enter an observable only in connection with another chirally odd distribution or fragmentation function, and due to twist-3 its contribution is suppressed by a factor of M_N/Q, where Q denotes the hard scale of the process. e.g. the combination $\sum_q e_q^2 \, e^q(x) \, e^{\bar{q}}(x)$ contributes to the Drell-Yan process with unpolarized proton beams, but only at twist-4 and together with other twist-4 quark-gluon-correlation functions [1, 2]. For some time this was the only known process involving $e^a(x)$, which of course is inpractical to access $e^a(x)$ experimentally.

Then the chirally and T-odd "Collins fragmentation function" $H_1^{\perp a}(z)$ has been introduced [35, 36], which describes the left-right asymmetry in the fragmentation of a transversely polarized quark of flavour a into a hadron. $H_1^{\perp a}(z)$ is "twist-2" in the sense that its contribution to observables is not power suppressed. Assuming factorization for transverse

[8] We thank the referee for pointing out this to us.

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momentum dependent processes, it was shown that the combination $\sum_q e_q^2\, e^q(x)\, H_1^{\perp q}(z)$ gives the dominant (tree-level) contribution to a single spin asymmetry, $A_{LU}^{\sin\phi}$, in hadron production from semi-inclusive DIS of longitudinally (subscript $_L$) polarized electrons off unpolarized (subscript $_U$) protons [36, 37, 38]. $A_{LU}^{\sin\phi}$ is proportional to the sine of the azimuthal angle ϕ of the produced pion around the z-axis defined by the exchanged virtual photon. This azimuthal asymmetry has been measured in the HERMES experiment and found consistent with zero within error bars [39, 40]. More recently, however, the CLAS collaboration reported the measurement of a non-zero $A_{LU}^{\sin\phi}$ in a different kinematics [41, 42].

In the HERMES experiments also another azimuthal asymmetry, $A_{UL}^{\sin\phi}$, in pion production from semi-inclusive DIS of unpolarized positrons off a longitudinally polarized proton target has been studied and found to be sizeable for π^+ and π^0 [39, 40]. This asymmetry contains information on $H_1^{\perp a}(z)$ and the chirally odd twist-2 $h_1^a(x)$ and twist-3 $h_L^a(x)$ distribution functions [38]. Using the model predictions from refs. [43, 44] for $h_1^a(x)$ and $h_L^a(x)$, in ref. [45] the relation among the favoured Collins and unpolarized fragmentation functions $H_1^\perp(z) = (0.33 \pm 0.06)\, z\, D_1(z)$ for $0.2 \leq z \leq 0.7$ at $\langle Q^2 \rangle = 2.4\,\mathrm{GeV}^2$ has been extracted from the HERMES data [39, 40].

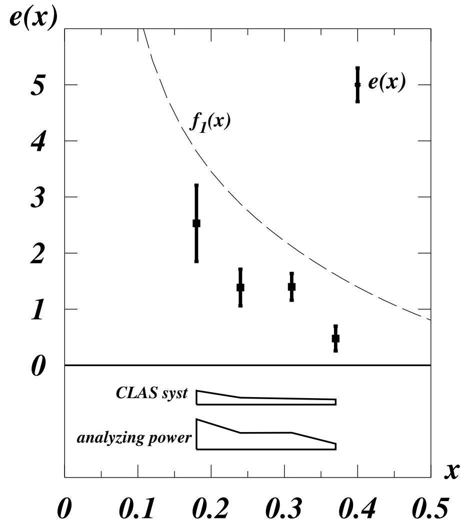

Figure 2: The $e(x) \equiv (e^u + \frac{1}{4}e^{\bar{d}})(x)$ at $\langle Q^2 \rangle = 1.5\,\mathrm{GeV}^2$ vs. x as extracted in [20] from the CLAS data [42]. For comparison is shown the corresponding flavour-combination of $f_1^a(x)$ (from [46]).

This result for $H_1^\perp(z)$ has been used in ref. [20] to extract first experimental information on the flavour combination $(e^u + \frac{1}{4}e^{\bar{d}})(x)$ from the CLAS data on $A_{LU}^{\sin\phi}$ in π^+ production [42]. For that it was assumed that the CLAS data is due to the Collins effect and the tree-level description of the process of refs. [37, 38] can be applied at the modest scale $\langle Q^2 \rangle = 1.5\,\mathrm{GeV}^2$ of the CLAS experiment. The extracted $(e^u + \frac{1}{4}e^{\bar{d}})(x)$ is definitely not small, about half the magnitude of the corresponding flavour combination of $f_1^a(x)$ in the region $0.15 \leq x \leq 0.4$ covered in the CLAS experiment, see figure 2. It will be interesting to see whether the first information on $e^a(x)$ reported in ref. [20] will be confirmed in future experiments.

The process described above is at present the cleanest way to access $e^a(x)$. However, there are other processes, e.g. electro-production of transversely polarized Λ from SIDIS of a longitudinally polarized electron beam off an unpolarized proton target, where $e^a(x)$ contributes together with further unknown fragmentation and distribution functions [38].

5. Conclusions

A brief discussion of the twist-3 chirally odd distribution function $e^a(x)$ has been given, focusing on theoretical properties, model estimates and perspectives to measure $e^a(x)$.

In particular, it has been emphasized that QCD equations of motion imply the existence of a δ-type singularity in $e^a(x)$ at $x = 0$. The first Mellin moment of $e^a(x)$ is solely due to this $\delta(x)$-contribution. This means that unfortunately DIS-experiments will not provide any information on the phenomenologically interesting pion nucleon sigma-term, which formally is related to the first moment of $e^a(x)$. Historically these conclusions have been drawn first on the basis of Regge arguments before the advent of QCD [22].

The existence of the $\delta(x)$-function, however, can indirectly be confirmed in the experiment by observing that the first moment of $e^a(x)$ vanishes if the point $x = 0$ is not included in the x-integration. If current quark mass effects are neglected (which practically are suppressed by m_q/Q), $e^a(x)$ satisfies the following "practical DIS-sum rules"

$$\int_{0+}^{1} dx\ (e^q + e^{\bar q})(x) \approx 0\,, \qquad \int_{0}^{1} dx\ x\,(e^q - e^{\bar q})(x) \approx 0\,. \tag{5.1}$$

The integration limit 0+ in the sum rule for the first moment means that the point $x = 0$ is not included. This corresponds to the experimental situation, since data can be obtained only for $x \geq x_{\min} > 0$, with $x_{\min}$ depending on the facility. In principle the vanishing of the second moment of $e^a(x)$ could also be tested experimentally by taking the difference between "structure functions" $\sum_a e_a^2\, e^a(x)$ extracted from proton and neutron data, and assuming a flavour symmetric sea, i.e. the relation $\int_0^1 dx\, x(e^{\bar u} - e^{\bar d})(x) = 0$. (However, in the case of $f_1^{\bar q}(x)$ an analogue relation, the "Gottfried sum rule" turned out to be wrong: $\int_0^1 dx\,(f_1^{\bar u} - f_1^{\bar d})(x) \neq 0$ [48].)

The "practical" sum rules (5.1) recall the situation of the twist-3 distribution function $\overline{g}_2(x) = g_2(x) - g_2^{WW}(x)$. The first moment of $\overline{g}_2(x)$ vanishes due to the Burkhardt-Cottingham sum rule [26] and the second moment due to the Efremov-Teryaev-Leader-sum rule [47]. In the case of the chirally odd distribution $e^a(x)$ the sum rules (5.1) will be even more difficult to test in the experiment.

In the limit of a large number of colours N_c one finds $e^u(x) = e^d(x)$ modulo $1/N_c$ corrections, and in the non-relativistic one obtains $e^q(x) = f_1^q(x)$ modulo relativistic corrections. Both relations could serve as useful guidelines. In relativistic models, such as the bag [2, 31], spectator [32] and chiral quark-soliton model (χQSM) [17], $e^a(x)$ has a sizeable valence-like structure at low scales about 0.5 GeV and is roughly half the magnitude of $f_1^a(x)$. These models do not respect the practical DIS sum rules (5.1) since the latter follow from the QCD equations of motions — but in the bag and chiral quark-soliton model different (model-) equations of motions hold. Still, in the χQSM there is a $\delta(x)$ in $e^q(x)$ [17, 33]. The $\delta(x)$-function arises in the χQSM from long-distance quark-quark correlations [33] — i.e. from basically the same non-perturbative mechanism found previously in the Gross-Neveu model in $(1+1)$-dimensions [25].

Experimentally $e^a(x)$ could be accessed by means of the Collins effect [35] in semi-inclusive DIS of polarized electrons off an unpolarized target [37, 38]. Recently the CLAS collaboration [41, 42] studied the process $\vec{e}p \to \pi^+ X$ and observed a particular angular distribution of the produced pions in the single (beam) spin asymmetry — as one would expect on grounds of the Collins effect [35, 37, 38]. If this interpretation, that the CLAS data [41, 42] are due to the Collins effect, applies then $e^a(x)$ is definitely not small, roughly half the magnitude of $f_1^a(x)$ at a scale of about $1.5\,\mathrm{GeV}^2$ [20].

The CLAS experiment could provide further insights, as could possibly do other fixed target experiments such as HERMES and COMPASS — when focusing on the large z region where the analyzing power $H_1^\perp(z)/D_1(z)$ is larger [45]. Also EMC at CERN (by a reanalysis of old data) and HERA at DESY could provide further information — where the advantage of a polarized lepton beam and an unpolarized proton could be used to explore especially the small x-region at moderate Q^2 needed to resolve the twist-3 effect.

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A. Consequences from using equations of motions

In this appendix the consequences are explored of using the identity in eq. (2.12) based on the equations of motions. The Mellin moments are defined as $\mathcal{M}_n[q] = \int_{-1}^1 dx\, x^{n-1} q(x)$.

The δ-function. The singular contribution originates from the local term $\bar\psi_q(0)\psi_q(0)$ in the identity (2.12) and is given by

$$e^q_{\rm sing}(x) = \frac{1}{2M_{\rm N}} \int \frac{d\lambda}{2\pi}\, e^{i\lambda x}\, \langle N|\bar\psi_q(0)\psi_q(0)|N\rangle = \delta(x)\, \frac{1}{2M_{\rm N}}\, \langle N|\bar\psi_q(0)\psi_q(0)|N\rangle\,, \qquad \text{(A.1)}$$

with the Mellin moments $\mathcal{M}_n[e^q_{\rm sing}] = \delta_{n1}\, \frac{1}{2M_{\rm N}}\, \langle N|\bar\psi_q(0)\psi_q(0)|N\rangle$.

The pure twist-3 term. The second term on the right hand side (r.h.s.) of the identity (2.12) gives rise to

$$\begin{aligned} e^q_{\rm tw3}(x) &= \frac{1}{4M_{\rm N}} \int \frac{d\lambda}{2\pi}\, e^{i\lambda x}\, \lambda^2\, \mathcal{F}^q_{\rm tw3}(\lambda)\,, \qquad \text{(A.2)} \\ \mathcal{F}^q_{\rm tw3}(\lambda) &= \int_0^1 du \int_0^u dv\, \langle N|\, \bar\psi_q(0)\, \sigma^{\alpha\beta}\, n_\beta\, [0, v\lambda n]\, gG_{\alpha\nu}(v\lambda n)\, n^\nu\, [v\lambda n, u\lambda n]\, \psi_q(u\lambda n)\, |N\rangle\,. \end{aligned}$$

For the Mellin moments one obtains (using the support property $e^a(x) \equiv 0$ for $|x| \geq 1$)

$$\begin{aligned} \mathcal{M}_n[e^q_{\rm tw3}] &= \frac{1}{4M_{\rm N}} \int_{-\infty}^{\infty} dx\, x^{n-1} \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi}\, e^{i\lambda x}\, \lambda^2\, \mathcal{F}^q_{\rm tw3}(\lambda) \\ &= \frac{1}{4M_{\rm N}} \int_{-\infty}^{\infty} d\lambda\, \lambda^2\, \mathcal{F}^q_{\rm tw3}(\lambda) \left(\frac{\partial}{i\partial\lambda}\right)^{n-1} \int_{-\infty}^{\infty} \frac{dx}{2\pi}\, e^{i\lambda x} \end{aligned}$$

$$= \frac{1}{4M_{\rm N}} \left(- \frac{\partial}{i\partial\lambda} \right)^{n-1} \lambda^2 \, \mathcal{F}^q_{\rm tw3}(\lambda) \Big|_{\lambda=0} . \tag{A.3}$$

From eq. (A.3) we immediately see that the first two moments of $e^q_{\rm tw3}(x)$ vanish

$$\mathcal{M}_1[e^q_{\rm tw3}] = \mathcal{M}_2[e^q_{\rm tw3}] = 0 . \tag{A.4}$$

Higher Mellin moments, $n > 2$, are generally non-zero. For explicit expressions see [3, 4, 5].

The mass term. The mass term follows from the third operator on the r.h.s. of the identity (2.12)

$$e^q_{\rm mass}(x) = - \frac{m_q}{M_{\rm N}} \int \frac{d\lambda}{4\pi} \, e^{i\lambda x} \, i\lambda \int_0^1 du \, \langle N | \, \bar\psi_q(0) \, \not{n} \, [0, u\lambda n] \, \psi_q(u\lambda n) | N \rangle . \tag{A.5}$$

Taking Mellin moments of $e^q(x)^{\rm mass}$ (and using the support property) yields

$$\begin{aligned}
\mathcal{M}_n[e^q_{\rm mass}] &= - \frac{m_q}{M_{\rm N}} \int_{-\infty}^{\infty} dx \, x^{n-1} \int_{-\infty}^{\infty} \frac{d\lambda}{4\pi} \, i\lambda \, e^{i\lambda x} \int_0^1 du \, \langle N | \, \bar\psi_q(0) \, \not{n} \, [0, u\lambda n] \, \psi_q(u\lambda n) | N \rangle \\
&= - \frac{m_q}{2M_{\rm N}} \int_{-\infty}^{\infty} d\lambda \, i\lambda \int_0^1 du \, \langle N | \, \bar\psi_q(0) \, \not{n} \, [0, u\lambda n] \, \psi_q(u\lambda n) | N \rangle \left(\frac{\partial}{i\partial\lambda} \right)^{n-1} \int_{-\infty}^{\infty} \frac{dx}{2\pi} \, e^{i\lambda x} \\
&= - \frac{m_q}{2M_{\rm N}} \int_0^1 du \left(- \frac{\partial}{i\partial\lambda} \right)^{n-1} i\lambda \, \langle N | \, \bar\psi_q(0) \, \not{n} \, [0, u\lambda n] \, \psi_q(u\lambda n) | N \rangle \Big|_{\lambda=0} \qquad \text{(A.6)} \\
&= \begin{cases} 0 & \text{for } n = 1 \\ \dfrac{m_q}{2M_{\rm N}} \displaystyle\int_0^1 du (n-1) \Big(i \frac{\partial}{\partial\lambda} \Big)^{n-2} \langle N | \bar\psi_q(0) \, \not{n} [0, u\lambda n] \psi_q(u\lambda n) | N \rangle \Big|_{\lambda=0} & \text{for } n \geq 2 . \end{cases}
\end{aligned}$$

Using the relation

$$\left(i \frac{\partial}{\partial\lambda} \right)^m [0, u\lambda n] \, \psi(u\lambda n) \Big|_{\lambda=0} = (i \, u \, n_\alpha D^\alpha)^m \psi(0)$$

in the expression for $n \geq 2$ yields

$$\begin{aligned}
&\frac{m_q}{2M_{\rm N}} \int_0^1 du \, (n-1) \left(i \frac{\partial}{\partial\lambda} \right)^{n-2} \langle N | \, \bar\psi(0) \, \not{n} \, [0, u\lambda n] \, \psi(u\lambda n) | N \rangle \Big|_{\lambda=0} = \\
&\qquad = \frac{m_q}{2M_{\rm N}} \langle N | \, \bar\psi(0) \, \not{n} \, (i \, n_\alpha D^\alpha)^{n-2} \, \psi(0) | N \rangle \int_0^1 du \, (n-1) \, u^{n-2}
\end{aligned}$$

such that we obtain

$$\mathcal{M}_n[e^q_{\rm mass}] = \begin{cases} 0 & \text{for } n = 1 \\ \dfrac{m_q}{2M_{\rm N}} \langle N | \, \bar\psi(0) \, \not{n} \, (i \, n_\alpha D^\alpha)^{n-2} \, \psi(0) | N \rangle & \text{for } n \geq 2 . \end{cases} \tag{A.7}$$

Recalling the definition of the twist-2 "unpolarized" distribution function $f_1^q(x)$ and its moments

$$
\begin{aligned}
f_1^q(x) &= \int \frac{d\lambda}{4\pi}\, e^{i\lambda x}\, \langle N|\, \bar\psi_q(0)\, \not{n}\, [0,\lambda n]\, \psi_q(\lambda n)\, |N\rangle\,, \qquad f_1^{\bar q}(x) = -f_1^q(-x)\\
\mathcal{M}_n[f_1^q] &= \frac{1}{2}\langle N|\, \bar\psi_q(0)\, \not{n}\, (i\, n_\alpha D^\alpha)^{n-1}\, \psi_q(0)\, |N\rangle
\end{aligned} \tag{A.8}
$$

we obtain the relation between the mass term and $f_1^q(x)$ quoted in eq. (2.16).

In the formal manipulations in eqs. (A.3), (A.6) it was assumed that the order of integrations over x and λ can be interchanged. In general this may not be allowed (see e.g. the discussion in section 5.4 of ref. [49]). However, in above cases one does not need to worry, because all moments of $e^a(x)$ are well defined (since $\sigma_{\pi\mathrm{N}}$, $N_q\frac{m_q}{M_\mathrm{N}}$, etc. are finite).

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